mines the frequencies and rates of decay corresponding to the Legendre function of order n is

$$\frac{\partial}{\partial a}\{a^{\frac{1}{2}}\mathrm{J}_{n+\frac{1}{2}}(\kappa a)\}\frac{\partial}{\partial b}\{b^{\frac{1}{2}}\mathrm{K}_{n+\frac{1}{2}}(\iota\kappa b)\}-\frac{\partial}{\partial a}\{a^{\frac{1}{2}}\mathrm{K}_{n+\frac{1}{2}}(\iota\kappa a)\}\frac{\partial}{\partial b}\{b^{\frac{1}{2}}\mathrm{J}_{n+\frac{1}{2}}(\kappa b)\}$$

$$=\frac{\iota\kappa\sigma}{4\pi\mathrm{V}}\left[\mathrm{J}_{n+\frac{1}{2}}(\kappa a)\frac{\partial}{\partial b}\{b^{\frac{1}{2}}\mathrm{K}_{n+\frac{1}{2}}(\iota\kappa b)\}-\mathrm{K}_{n+\frac{1}{2}}(\iota\kappa a)\frac{\partial}{\partial b}\{b^{\frac{1}{2}}\mathrm{J}_{n+\frac{1}{2}}(\kappa b)\}\right]$$

$$\frac{\partial}{\partial a}\{a^{\frac{1}{2}}\mathrm{J}_{n+\frac{1}{2}}(\kappa' a)\}\Big/\mathrm{J}_{n+\frac{1}{2}}(\kappa' a),$$

where $\kappa'^2 = -4\pi\iota\kappa\mathrm{V}/\sigma$, and the values of κ are complex quantities. It can be readily shown that when the conductivity of the inner sphere is of the same order as the conductivity of sea-water, the rates of decay associated with the frequencies that are of the order of those of the waves of wireless telegraphy are small when $b-a$ is small compared with b; it follows that the time required to establish an approximately steady state is impracticably long.*

A Comparison of Magnetic Declination Changes at British Observatories.

By C. Chree, Sc.D., LL.D, F.R.S.

(Received January 27, 1921.)

In spite of the very varied use of the compass needle for purposes of navigation and surveying, there is little really known about the closeness in the parallelism of daily magnetic changes in different parts of the United Kingdom. The mean annual values from magnetic observatories show that, since 1910, secular change of declination has been at least approximately the same throughout England, the south of Scotland, and the south-west of Ireland. Thus the conditions have been favourable for an enquiry into the parallelism of other changes.

Diurnal inequalities from five selected quiet days a month were published for Falmouth as well as Kew up to 1912. A comparison of results from a number of years combined had shown little difference between the diurnal inequalities at the two stations as regards the range. Difference in local time produced a visible effect, but it was small. In more recent years, corresponding diurnal inequalities for Eskdalemuir and Kew from the five international quiet days of each month had shown a close similarity.

---

* The rates of decay of electrical currents in an imperfectly conducting sphere have been investigated by Prof. H. Lamb, "Electrical Motions in Spherical Conductors," 'Phil. Trans.,' Part II, p. 530 (1883).

On the other hand, it had been found that, while a regular diurnal inequality of declination is derivable from a number of disturbed days combined, it differs markedly in type as well as in range from the quiet day inequality; also, comparisons of Kew and Eskdalemuir disturbed curves had shown a marked tendency for the latter to be the more disturbed station.

Various lines of investigation have been followed. A comparison has been made of diurnal inequalities at Eskdalemuir and Kew, based on days of different magnetic character, varying from the international quiet days of 1913–16 to groups of days respectively of characters 0 (quiet), 1 and 2 (highly disturbed) during several months of 1919, when a declination magnetograph was in operation at Eskdalemuir. The difference between the Eskdalemuir and Kew diurnal inequalities was found to increase with the amount of disturbance, and to be of the same general type as the difference between disturbed days' and quiet days' inequalities at Kew.

Next, an attempt was made to see how closely mean daily and mean hourly values at Kew and Eskdalemuir agreed when a constant correction was applied to the Kew value, representing the mean difference of declination at the two places. All such comparisons are necessarily affected by any base value uncertainty at either station.

The Eskdalemuir value being calculated from the Kew value in the way indicated above, the calculated mean for the average day of the several months considered was found to differ from that actually observed by about 0·5′. Any error in the assumed base line values would be likely to exaggerate this difference.

The variability in the case of the hourly values was measured by the range in the quantity D (declination) at Eskdalemuir—D at Kew, derived from the mean values for each of the twenty-four hours of the day. It was found to depend greatly on whether the day was quiet or disturbed, the mean ranges obtained being 2·1′ for character 0, 3·2′ for character 1, and 7·6′ for character 2 days. The months considered contained no disturbance of the first order, but on one day of character 2 the observed range in D at Eskdalemuir—D at Kew was no less than 32·9′. We may regard the half of the mean range as representing the worst mistake likely to be made on the average day of the magnetic character considered, in arriving at the declination appropriate to a field station in the south of Scotland from observations extending over an hour, when the reduction to the mean value for the day is dependent on magnetograms taken in the south of England.

When the observations in the field are limited, as is usually the case, to a few minutes, the uncertainty is naturally greater. But, even if such were not the case, it is obvious that, for high accuracy, field observations taken on days of character 2 had better be scrapped.

Oscillations of considerable size sometimes occur on days the greater part of which is very quiet, and they are often numerous on days when there is no very large range of declination. In a previous paper* dealing with Eskdalemuir (E), Stonyhurst (S), Falmouth (F), and Kew (K) curves for a good many months of 1908–9, I had found for the mean values of the ratios of the oscillation amplitudes

$$E/K = 2{\cdot}14\,;\ S/K = 1{\cdot}68\,;\ F/K = 1{\cdot}14.$$

The ratios derived from different oscillations, or even from the mean oscillation of a single day, showed a considerable range of values, but so marked was the pre-eminence of the Eskdalemuir and Stonyhurst oscillations that there was no single case in which the amplitude at Kew exceeded that at Eskdalemuir, and in only 1 out of 533 cases was the amplitude larger at Kew than at Stonyhurst.

This comparison was repeated for disturbances from June to December, 1919, at Eskdalemuir, Stonyhurst, and Kew, but with very different results. As before, there was no single day which gave a lower mean amplitude at Eskdalemuir than at Kew; but the mean value of E/K from 295 oscillations was only 1·54, the values of the ratio for individual days varying from 1·21 to 2·09.

There was an even larger reduction in the mean value of S/K, which was only 1·01. Also, while October gave a mean value of 1·16, the value falling below unity in only 2 out of 16 days, the months of July, August, and September combined gave a mean of only 0·89, and on only 1 out of 25 days did the ratio exceed unity.

In view of these unexpected results, a comparison was instituted between oscillations at Kew and at Falmouth, extending from January, 1908, to May, 1913, when registration ceased at Falmouth. In all, 2441 declination oscillations, occurring on 489 days, were measured. The mean value of F/K exceeded unity in 45 out of the 49 months from January, 1908, to January, 1912, the mean from the four complete years, 1908 to 1911, being 1·16. But the mean for 1912 was only 1·00, and only one month, from February, 1912, to April, 1913, gave a value in excess of 1.

It is clear that, whilst the ratio of the amplitudes of declination oscillations at two stations in the United Kingdom may show a marked approach to constancy for months at a time, it may show large variations over a period of years. It is largely dependent on circumstances other than the geographical co-ordinates of the stations. For a complete elucidation of the phenomena much further investigation is required, in which account must be taken of all the magnetic elements.

---

* 'Inst. of Electrical Engineers,' vol. 57, p. 591.

In the present enquiry the author is much indebted to the kind co-operation of the Rev. A. L. Cortie, S.J., and Dr. Crichton Mitchell, F.R.S.E. The former lent a number of Stonyhurst curves, while the latter supplied the Eskdalemuir curves with full particulars of the hourly values and base-line values.

A more complete discussion, with full particulars of the observational data, will be published in the 'Geophysical Memoirs' of the Meteorological Office.

Siren Harmonics and a Pure Tone Siren.

By E. A. Milne and R. H. Fowler.

(Communicated by Prof. A. V. Hill, F.R.S. Received November 9, 1920.)

§ 1. *Introduction.*—This paper deals with the purity of tone of the sound produced by an ordinary Seebeck siren, and with the problem of designing a siren to produce a pure, or comparatively pure, tone. The results were obtained in the course of certain technical experiments on the location of aircraft by sound; they seem, however, to be of more general interest, as a source of sound giving a fairly pure tone of some intensity, whose frequency may be varied at will, is not easily obtainable. The results here described were obtained by the authors in the course of their work as technical officers of the Munitions Inventions Department, and are published by permission of the Department. The experiments were carried out at the Acoustical Research Section of the Department with the help of Major W. S. Tucker, R.E. The authors' best thanks are due to Major Tucker and his staff for their assistance.

The Seebeck siren, as described in books on sound, consists essentially of a revolving disc bored with holes, against which a stream of air is driven from a delivery pipe. The holes are equally spaced round a circle, and the pipe is fixed so that as the disc revolves the holes pass the mouth of the pipe in succession; when no hole is opposite the pipe the flow of air is partially or completely interrupted, and as a hole approaches and passes the flow of air waxes and wanes. When the disc is rotated sufficiently fast, a musical note